Role of relativistic kinematics in describing two-quark systems

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Abstract

An attempt to incorporate relativistic kinematics in the description of light quark systems is made. It seems that the way of such incorporation along the suggestion expressed by R. Gaida and his collaborators is very promising. Comparison of these results with the experimental data concerning boson mass spectrum shows that this approach is among the best theoretical interpretation of the data.

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Surprising success of non-relativistic quark models with funnel or oscillator potential in describing hadron mass spectra for heavy quark systems had waken the hopes that at least some aspects of the strong interactions are understood after all. Good insight of previous works concerning this subject is given in [1, 2, 3]. But evidently the picture is not complete until it incorporates the light quarks u, d and s. But this means that the problem is shifting into quantum chromodynamical sector. In this respect the search of alternative possibilities of incorporating relativistic kinematics is quite actual. A very promising approach to the solution of relativistic problem of interaction of two particles was suggested recently by R. Gaida and his collaborators [4]-[7]. Their results can be directly applied to our problem of calculating the mass spectrum of quarkonium as a system of two-quarks. Somewhat different approach was simultaneously developed by I. Todorov and P. Bogolyubov [8, 9] and later by E. Predazzi et. al. [10]. In this work we shall investigate the results concerning the application of their findings to light-quark systems.

Let us follow first the way, suggested by Predazzi et al [10]-[12]. Following the ideas expressed in [10, 11] and later developed in [12], let us start with the classical expression for the relativistic total energy of two particle system with masses m and M, respectively

$$E = \sqrt{\mathbf{p}_1^2 + m^2} + \sqrt{\mathbf{p}_2^2 + M^2} \tag{1}$$

were $\mathbf{p}_1 = -\mathbf{p}_2 \equiv \mathbf{p}$ in the c.m. system. The main problem here is the mode of inclusion the interaction between two particles. For equal masses Lucha and Schoberl [13] write basing on (1) and passing to quantum mechanics

$$\left(2\sqrt{\mathbf{p}^2 + m^2} + V\right)\Psi\left(\mathbf{r}\right) = E\Psi\left(\mathbf{r}\right) \tag{2}$$

which they call the spinless Bethe-Salpeter equation. They solve very cunningly this equation in the configurational momentum representation but obtain not very good results, which we shall discuss later. But in any case we can conclude that exactly the way of introducing the interaction is responsible for their failure. Therefore we propose here another way of introducing the interaction. Namely, following Predazzi et al [12] we linearize the expression

$$\left(\frac{E^2 + m^2 - M^2}{2E}\right)^2 = \mathbf{p}^2 + m^2 \tag{3}$$

which follows from (1) after simple algebraic transformations, and obtain

$$\left(\frac{E^2 + m^2 - M^2}{2E}\right)\Psi\left(\mathbf{r}\right) = (\alpha c\mathbf{p} + \beta mc^2 + \widetilde{V})\Psi\left(\mathbf{r}\right) \tag{4}$$

where α and β are the usual Dirac matrices, Ψ is the four- component wave function for which we shall use two-component representation

$$\Psi\left(\mathbf{r}\right) = \begin{pmatrix} \varphi\left(\mathbf{r}\right) \\ \chi\left(\mathbf{r}\right) \end{pmatrix}. \tag{5}$$

In the general case we would have on this stage to decide what kind of Lorentz-transform properties we shall ascribe to the interaction.

In general the interaction can transform either like Lorentz-scalar (like the mass) or to be a 4-th component of Lorentz-vector, i.e. transform like energy.Then we shall consider the interaction $\widetilde{V}$ as a mixture

$$\widetilde{V} = \beta S \cdot (1-\varepsilon) + \mathbf{I}V \cdot \varepsilon \tag{6}$$

where ε is some mixing parameter. In what follows we shall simplify the expression for $\widetilde{V}$ by taking $\varepsilon = 1/2$ (which is suggested by experimental data (see,e. g. [14] and also [15]) what means that (6) can be written in the form

$$\widetilde{V} = \frac{1}{2}(\beta + \mathbf{I})\,(V+S)\,. \tag{7}$$

This form is chosen in order to obtain simple non-relativistic result $\widetilde{V} = V$ if there is no difference between S and V. Such potential was introduced previously by V. Kukulin, M. Moshinsky et. al. [16, 17] and was called an averaged potential, which allows to reduce the system of Dirac equations to a single relativistic oscillator equation.In this, particular case when the mixture of scalar S and vector V part of potentials are equal we shall consider the following possibilities

$$S(r) + V(r) \equiv \widetilde{V} = -\frac{\alpha_s}{r} + A \cdot r^2 + V_0, \tag{8}$$

$$S(r) + V(r) \equiv \widetilde{V} = -\frac{\alpha_s}{r} + k \cdot r + V_0, \tag{9}$$

$$S(r) + V(r) \equiv \widetilde{V} = \frac{g^2}{6\pi\mu}\left(1 - e^{-\mu r}\right) - \frac{16\pi}{25}\frac{e^{-kr}}{r \cdot \ln\left(b + \left(\frac{1}{\Lambda r}\right)^2\right)} + V_0. \tag{10}$$

Today all of these potentials are used to describe the quark-antiquark interaction. The discussion concerning the advantages and handicapes of these potentials is given in [18]. Our aim is to apply these potentials to describing the meson mass spectra with relativistic kinematics which is built into equation (4). In comparison to [18] the search of the best description by minimizing χ^2 will be given. For an averaged potential (7) the equation (4) reduces to single equation for the large wave function $\varphi(\mathbf{r})$

$$(E-V)^2\,\varphi(\mathbf{r}) = \left[4\mathbf{p}^2 + 4m^2 + 4mS + S^2\right]\varphi(\mathbf{r}) = \left[4\mathbf{p}^2 + 4\left(m + \frac{S}{2}\right)^2\right]\varphi(\mathbf{r})\,. \tag{11}$$

For this propose we have to solve numerically the following equation

$$[\mathbf{p}^2 + \left(\frac{E}{2} + m\right) \cdot \frac{\widetilde{V}}{2} - \left(\frac{E^2}{4} - m^2\right)]\varphi(\mathbf{r}) = 0. \tag{12}$$

Transferring to operators and carrying out the substitutions for unknown function

$$\varphi(\mathbf{r}) = \frac{\Phi(\mathbf{r})}{\mathbf{r}} \tag{13}$$

one obtains the equation

$$\left[\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} - \left(\frac{E}{2} + m\right) \cdot \frac{\widetilde{V}}{2} + \frac{E^2}{4} - m^2\right] \Phi(\mathbf{r}) = 0. \tag{14}$$

In a simple approximation

$$S + V = \frac{1}{2}\left(Ar^2 + V_0\right) \tag{15}$$

one obtains

$$[\mathbf{p}^2 + \left(\frac{E}{2} + m\right)\left(\frac{1}{2}Ar^2 + \frac{1}{2}V_0\right) - \left(\frac{E^2}{4} - m^2\right)]\varphi(\mathbf{r}) = 0 \tag{16}$$

which leads to the equation for relativistic isotropic oscillator

$$\left[\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} - \left(\frac{E}{2} + m\right) \cdot \frac{1}{2}Ar^2 + \frac{E^2}{4} - m^2 - \left(\frac{E}{2} + m\right) \cdot \frac{1}{2}V_0\right] \Phi(\mathbf{r}) = 0. \tag{17}$$

Now with the standard change of variables

$$\sqrt{\left(\frac{E}{2} + m\right)\frac{1}{2}A} \cdot r^2 = x^2 \tag{18}$$

one obtains

$$\left[\frac{d^2}{dx^2} - \frac{l(l+1)}{x^2} - x^2 + \frac{E^2/4 - m^2 - \frac{1}{2}V_0\left(\frac{E}{2} + m\right)}{\sqrt{\left(\frac{E}{2} + m\right)\frac{A}{2}}}\right] \Phi(\mathbf{r}) = 0. \tag{19}$$

The physical solution of (7) is satisfied, when

$$\frac{E^2}{4} - m^2 = \sqrt{\frac{A}{2} \cdot \frac{E + 2m}{2}} \cdot (4N + 2l + 3) + \frac{V_0\,(E/2 + m)}{2} \tag{20}$$

or

$$\frac{E}{2} - m = \sqrt{\frac{A}{E + 2m}}\,(4N + 2l + 3) + \frac{V_0}{2} \tag{21}$$

In the non-relativistic limit when $E \sim 2m$ one has

$$E \approx 2m + \sqrt{\frac{A}{m}}\,(4N + 2l + 3) + V_0 \tag{22}$$

in full accordance with the non-relativistic case (see,e.g.[18]).

Actually for large total energies E from (21) it follows $E^2 \sim l^{4/3}$ i.e. almost linear Regge-trajectory as it should be from general considerations. As we shall see the application of other variants of potentials gives even better results.

As we have mentioned earlier, recently very promising π approach to the problem of relativistic description of many-particle system was elaborated along with the ideas

presented by Gaida in [4] and elaborated by Tretyak, Spytko and Duviryak [5, 6, 7]. They used the Weyl quantization method and succeeded in solving the problem for relativistic oscillator between two particles.

Considering oscillator-type interaction

$$V = \omega^2 \mathbf{p}_1 \mathbf{p}_2 r^2 \tag{23}$$

where

$$\mathbf{p}_1 \mathbf{p}_2 = \frac{m_{RED}}{2} = \frac{m_q}{4} \tag{24}$$

the non-relativistic approximation they would obtain for mass of two-particle system

$$M = \left\{ \left[2m_q + \frac{\omega}{2} (4N + 2l + 3) \right]^2 + \frac{\omega^2}{4} \right\}^{1/2} + V_0 \tag{25}$$

or

$$M = \sqrt{\left(\sum m_q + \sqrt{\frac{A}{m_q}} \cdot (4N + 2l + 3) \right)^2 + \frac{A}{m_q}} + V_0 \tag{26}$$

if we express the (26) in form of string tension A and generalize their results to our boundary condition of isotropic oscillator.

Exactly such approximation was taken in [18]. But if we suggest that in general

$$\mathbf{p}_1 \mathbf{p}_2 = \frac{E + m_q}{8} \tag{27}$$

then Lvov group results

$$M = \left\{ \left[2m_q + \sqrt{\frac{2A}{E + m_q}} \cdot (4N + 2l + 3) \right]^2 + \frac{2A}{E + m_q} \right\}^{1/2} + V_0 \tag{28}$$

will resemble very much our relativistic approach. The results of calculations according to (28) are given below. The parameters are taken to be $A = 0.01\ GeV^3$, $V_0 = -0.436\ GeV$ certainly this approximation is valid if one considers the energy dependence of $\mathbf{p}_i$ only on the last stage of calculations.

It is interesting that similar to (28) result for $\mathbf{M^2}$- operator was obtained in relativistic approach by Ishida-Oda based on special assumption of covariant relativistic approach. Even the numerical values of parameters of Ishida-Oda [19] are close to the result of (28). Their value $A = 0.05 GeV^3$ which is of the same order as our A. More precise comparison is impossible because of ambiguity of other parameters.

Usually such potential models like we have used here are called naïve quark model. But exactly our model is not-so naïve. Firstly, the relativistic kinematics not only renders it more complicated, but shows the possible way of building the model of interaction of two relativistic particles. Secondly the potentials like (8-10) incorporate asymptotic freedom the strong coupling constant α_s was calculated according to the classical expression

$$\alpha_s (r) = \frac{12\pi}{33 - 2N} \cdot \frac{1}{\ln \left(\frac{1}{r^2 \widetilde{\Lambda}^2} \right)} \tag{29}$$

where $\widetilde{\Lambda}$ was taken to be equal to $\widetilde{\Lambda} = 0.14GeV$. And lastly the model allows to include spin-spin interaction either by passing to Breit-Fermi equation or by using Dirac equation straightly. In the Table 1 α_s is taken exactly according to (29).

It is interesting to note that the definition of mass can be given in a different way. Considering one particle as moving in the field of another and vice versa and adding the obtained masses we obtain the results with parameters which correspond very closely to our Table 1. Namely one can write down the Dirac equation for one particle moving in the outer field, reduce it to the equation for large component $\chi(\mathbf{r})$ and obtain [20]

$$\left(E^2 - m^2\right)\chi(\mathbf{r}) = \mathbf{p}^2\chi(\mathbf{r}) + (E+m)\,V\chi(\mathbf{r})\,. \tag{30}$$

Applying the virial theorem to this equation

$$\left\langle \chi(y)\left|\mathbf{p}^2(y)\right|\chi(y)\right\rangle = \frac{1}{2}\left\langle \chi(y)\left|y\frac{\partial V}{\partial y}\right|\chi(y)\right\rangle = \left\langle \chi(y)\left|y^2\right|\chi(y)\right\rangle \tag{31}$$

Combining (30) and (31) for oscillator interaction one obtains

$$E^2 - m^2 = 2\sqrt{C\,(E+m)}B_{Nl} + (E+m)\,V_0 \tag{32}$$

where

$$B_{Nl} = \left\langle \chi(y)\left|y^2\right|\chi(y)\right\rangle = 2N + l + 3/2 \tag{33}$$

This expression on resembles very much the (28). So we are left with the five most realistic from our point view possibilities, namely, of calculating masses according to (8), (9), (10), (26), (28).

To obtain the masses of multi-quark system according to these expressions, it is necessary to define the values of parameters. The masses of quarks were taken to be as usually in quark models $m_u = 0.33GeV$, $m_c = 1.675 - 1.75GeV$, $m_b = 5.05 - 5.1GeV$. Parameters for different potentials are shown in Table 1.

Table 1. The parameter values for different potentials

Potential	α_u	α_c	α_b	k, A	V_0, GeV
(8)	0.5	0.325	0.3	$k = 0.27$ (GeV^2)	-0.8356
(9)	0.5	0.386	0.3	$A = 0.01$ (GeV^3)	$(-0.436)...$ $...(-0.527)$
(10)	$\frac{g^2}{6\pi} = 0.3795$ (GeV^2)	$\mu = 0.054$ (GeV)	$K = 0.75$ (GeV)	$\Lambda = 0.35$ (GeV) $b = 4$	-1.103

Table 2.Variation of χ^2 for different parameters of potential (9).

k (GeV^2)	$\chi^2_{u\overline{u}}$	$\chi^2_{c\overline{c}}$	$\chi^2_{b\overline{b}}$	V_0 (GeV)
0.1826	514	$1.15 \cdot 10^5$	$1.3 \cdot 10^4$	-0.628
0.25	98	$6.2 \cdot 10^3$	$2 \cdot 10^4$	-0.791
0.26	67	$2.2 \cdot 10^4$	$1.6 \cdot 10^4$	-0.813
0.27	43.6	$3 \cdot 10^4$	$2.9 \cdot 10^3$	-0.835
0.29	12.6	$7 \cdot 10^4$	$8.5 \cdot 10^3$	-0.878
0.305	1.8	$1.36 \cdot 10^5$	$4.3 \cdot 10^4$	-0.908

The value of V_0 reflects the fact that potential is the Fourier-transform of scattering amplitude V_0 being the constant of interaction. Gromes [21] has evaluated this constant for linear confinement and obtained the value of V_0

$$V_0 \simeq -2\sqrt{k} \cdot e^{-(\gamma - 0.5)} \tag{34}$$

where k is the string tension $\gamma = 0.57721...$ is Euler-MacLoraint constant. According to our values of $k = 0.18 - 0.305 GeV^2$. According to (34) has to vary within the limits $V_0 = -(0.77 \div 1.02)\, GeV$ which is quite close to values, cited in the table (2). Let us stress that the set values are not colour-dependent, which, reduces the number of adjustable parameters.

The result of calculation together with experimental data are shown in Tables (3-6). Experimental values were taken from [22]. For choosing the parameters the minimum of χ^2-criterion was used, with the definition of χ^2 given in [18]. In this definition $\mathbf{N}$ is the number of meson masses, $\mathbf{n}-$ is the number of parameters (in our case we considered them to be equal to two namely confinement parameter and V_0), Δ is the experimental error in definition of experimental mass M_{EXP} of two-quark system [22] since V_0 was chosen to match the experimental value of ground-state mass we are actually left only which one adjustable parameter A (or k). Since we do not include LS-forces we had to take the average center of gravity (COG) value of P-resonances, which was calculated according to formula

$$M_{COG} = \frac{\sum_j (2j+1) \cdot M_j}{\sum_j (2j+1)}. \tag{35}$$

As one can see from both radial and orbital excitation calculation the variant which incorporates in one or another way the relativistic kinematics give better description of Regge-trajectories, which, are believed to be linear in l for M^2.

It is well known that even in non-relativistic limit one can obtain good description of $u\overline{u}$-$d\overline{d}$ -systems at the account of spoiling the $c\overline{c}$ of $b\overline{b}$ -description.

Indeed in the Table 2 we show χ^2 obtained for Eichten parameters. Due to a high precision of defining J/Ψ and Υ-mesons the large χ^2 were considered good for these mesons and bad for ρ-meson trajectory. On other hand the non-relativistic Badalyan [23] results are good for $u\overline{u}$, but bad for J/Ψ. Fabre [24] in order to obtain good results for light quarkonium had to change the very potential. Instead of this situation our way or incorporating the relativistic kinematics we obtain good results for all data. The table 2 demonstrates this statement. Table 3 contains the comparison of different potentials.

The discussion concerning the choice of potentials is given in [18]. We shall choose in what follows the Cornell-potential (9) which seems to be preferable though potential (10) is also quite good. The parameters α_s here are taken from table 1. The $u\overline{u}$-data are fantastically good, but $c\overline{c}$ and $b\overline{b}$-data could be better.Therefore in tables 4-6 we give the results for $k = 0.29GeV^2$. We consider these results as the best. It is interesting that the values which give these best results are close to those of Lucha and Schöberl [3].We want to stress that all the above results are obtained by numerical solution of (14). The last columns in $(4-6)$ are calculated according to (28). With the choice $A = 0.071GeV^3$, $V_0 = -1.0077GeV$, $m_u = 0.33GeV$, $m_c = 1.75GeV$, $m_b = 5.13GeV$ the results are quite comparable with the other entries. But still we have to conclude that pure oscillator potential is too rough to give final result. The use of more sophisticated potential is to be taken hear too. But it demonstrates nicely that inclusion of relativistic kinematics is very crucial.

We would like to call the attention to one interesting feature of relativistic models, namely that the slope of linear (or close to linear) Regge-trajectory in this case is constant, while in nonlinear models it is neither constant nor linear. Roughly experiments show this slope to be equal $\sim 1.2GeV^2$. In our cases it varies from 1.15 to $2.5GeV^2$ for different cases. As Tutik et [25] have indicated the Regge trajectories for low-lying states coincide while for large values of orbital momentum l the screened potential (10) leads a limited Regge-trajectory in contrast to infinitely rising trajectories for other potentials like (8) or (9).

Table 3. Mass spectrum of $u\overline{u}$-system with some realistic potentials.

State	*designation*	M_{EXP} GeV	$M_{TH}(8)$ GeV	$M_{TH}(9)$ GeV	$M_{TH}(10)$ GeV
$1S$	$\rho\ 1^+(1^{--})$	0.768 ± 0.0005	0.768	0.768	0.768
$1P$	${}^3P^*_{COG}$	1.262 ± 0.03	1.26	1.296	1.3
$2S$	$\rho\ 1^+(1^{--})$	1.465 ± 0.025	1.609	1.573	1.561
$2P$	$a_2\ 1^-(2^{++})$	1.935 ± 0.015	2.003	1.932	1.889
$3S$	$\rho\ 1^+(1^{--})$	2.15 ± 0.01	2.3	2.155	2.103
$1D$	$\rho_3\ 1^+(3^{--})$	1.691 ± 0.013	1.665	1.689	1.687
$2D$	$\rho_3\ 1^+(3^{--})$	2.25 ± 0.01	2.348	2.235	2.1845
$1F$	$a_4\ 1^-(4^{++})$	2.037 ± 0.036	2.03	2.021	2.003
$1G$	$\rho_5\ 1^+(5^{--})$	2.350 ± 0.015	2.366	2.312	2.275
$1H$	$a_6\ 1^-(6^{++})$	2.45 ± 0.13	2.685	2.576	2.514
χ^2	$-$	$-$	53.3	1.8	19.6

Table 4. The best mass spectrum of $u\overline{u}$-system with Cornell-potential (9) in formula (28).

State	*designation*	M_{EXP} GeV	$M_{TH}(9)$ GeV	$M_{TH}(28)$ GeV
$1S$	$\rho\ 1^+(1^{--})$	0.768 ± 0.0005	0.768	0.768
$1P$	${}^3P^*_{COG}$	1.262 ± 0.03	1.281	1.198
$2S$	$\rho\ 1^+(1^{--})$	1.465 ± 0.025	1.551	1.577
$2P$	$a_2 1^-(2^{++})$	1.935 ± 0.015	1.902	1.923
$3S$	$\rho\ 1^+(1^{--})$	2.15 ± 0.01	2.12	2.244
$1D$	$\rho_3\ 1^+(3^{--})$	1.691 ± 0.013	1.665	1.577
$2D$	$\rho_3\ 1^+(3^{--})$	2.25 ± 0.01	2.197	2.244
$1F$	$a_4\ 1^-(4^{++})$	2.037 ± 0.036	1.987	1.923
$1G$	$\rho_5\ 1^+(5^{--})$	2.350 ± 0.015	2.278	2.244
$1H$	$a_6\ 1^-(6^{++})$	2.45 ± 0.13	2.529	2.547
χ^2	–	–	12.6	35.68

Table 5. The same for $c\overline{c}$-system.

State	*designation*	M_{EXP} GeV	$M_{TH}(9)$ GeV	$M_{TH}(28)$ GeV
$1S$	$J/\Psi\ 0^-(1^{--})$	3.096 ± 0.00009	3.07	3.014
$1P$	$\chi_{cl}\ 0^+(1^{++})$	3.51 ± 0.00012	3.5118	3.331
$1D$	$\Psi\ ?^?(1^{--})$	3.770 ± 0.0025	3.837	3.632
$2S$	$\Psi\ 0^-(1^{--})$	3.688 ± 0.0001	3.732	3.632
$2D$	$\Psi\ ?^?(1^{--})$	4.159 ± 0.02	4.296	4.195
$3S$	$\Psi\ ?^?(1^{--})$	4.04 ± 0.01	4.227	4.195
$3D$	$\Psi\ ?^?(1^{--})$	4.415 ± 0.006	4.692	4.717
χ^2	–	–	$7 \cdot 10^4$	$8.4 \cdot 10^5$

Table 6. The same for $b\overline{b}$-system.

State	*designation*	M_{EXP} GeV	$M_{TH}(9)$ GeV	$M_{TH}(28)$ GeV
$1S$	$\Upsilon\ ?^?\ (1^{--})$	9.460 ± 0.00022	9.479	9.542
$1P$	${}^3P^*_{COG}$	9.892 ± 0.0007	9.883	9.741
$2S$	$\Upsilon\ ?^?(1^{--})$	10.023 ± 0.00031	10.037	9.932
$2P$	${}^3P^*_{COG}$	10.268 ± 0.00057	10.299	10.121
$3S$	$\Upsilon\ ?^?(1^{--})$	10.355 ± 0.0005	10.433	10.308
$4S$	$\Upsilon\ ?^?(1^{--})$	10.58 ± 0.0035	10.776	10.675
$5S$	$\Upsilon\ ?^?(1^{--})$	10.865 ± 0.008	11.062	11.034
$6S$	$\Upsilon\ ?^?(1^{--})$	11.019 ± 0.008	11.332	11.385
χ^2	–	–	$8.5 \cdot 10^3$	$7.4 \cdot 10^4$

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